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FREEZE DRYING - THEORETICAL CONSIDERATIONS AND APPLICATION

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↙ The principles governing freeze-drying rates of heat and mass transfer have been reviewed and defined in practical terms.

The various methods and resistances to heat transfer are discussed and practical use is made of the equations presented.

Mass transfer through the various phase changes is discussed. Theoretical discussions of the mode of mass transfer at each step and suitable relationships are developed which describe the limiting conditions for the mass transfer rates. By use of the over-all mass transfer equations, rates of drying and total times required may be determined.

Sample calculations illustrating the use of the relationships and theoretical considerations are included.

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I. INTRODUCTION

Freeze-drying is a method of drying materials by sublimation, using low temperatures and reduced pressures. It is generally employed where heat-sensitive materials must be processed to a dry state. The principle used is one in which the material to be dried is frozen in some convenient form; slabs or pellets are most frequently used. The frozen material is then subjected to a high vacuum which is below the vapor pressure of water at the temperature of the material to be sublimed. Under these conditions, the ice (or frozen material) sublimes from the frozen mass and is collected on a cooled condenser whose temperature is such that the vapor is desublimed. As sublimation continues, heat is withdrawn from the material being dried. In order to maintain the temperature of the material high enough so that the sublimation process will not stop, heat is supplied to the material from an evaporator. The rate at which heat is supplied is dependent on the drying cycle desired.

A typical apparatus consists of a horizontal, cylindrical body containing a horizontal evaporator and condenser. The evaporator and condenser consist of plates or coils in which a suitable heating and cooling medium may be circulated. Cast metal with a high thermal conductivity and a blackened surface to increase transfer of heat by radiation is used as material of construction for the trays which hold the product to be dried. The material trays may have ribs cast as an integral part, to provide more uniform heat transfer through the layer of material being dried. Sources of refrigeration, heat, and vacuum are connected to the dryer in order to provide proper programming of heat transfer.

Figure 1 shows a typical installation, including the arrangement of the basic components of the batch system. This report covers the theory of freeze-drying by the batch method, but does not cover theory specific to continuous systems such as that described by Maister.^{1/*}

* See Literature Cited.

— FLOW BELOW 200μ Hg VACUUM
 --- FLOW ABOVE 200μ Hg VACUUM

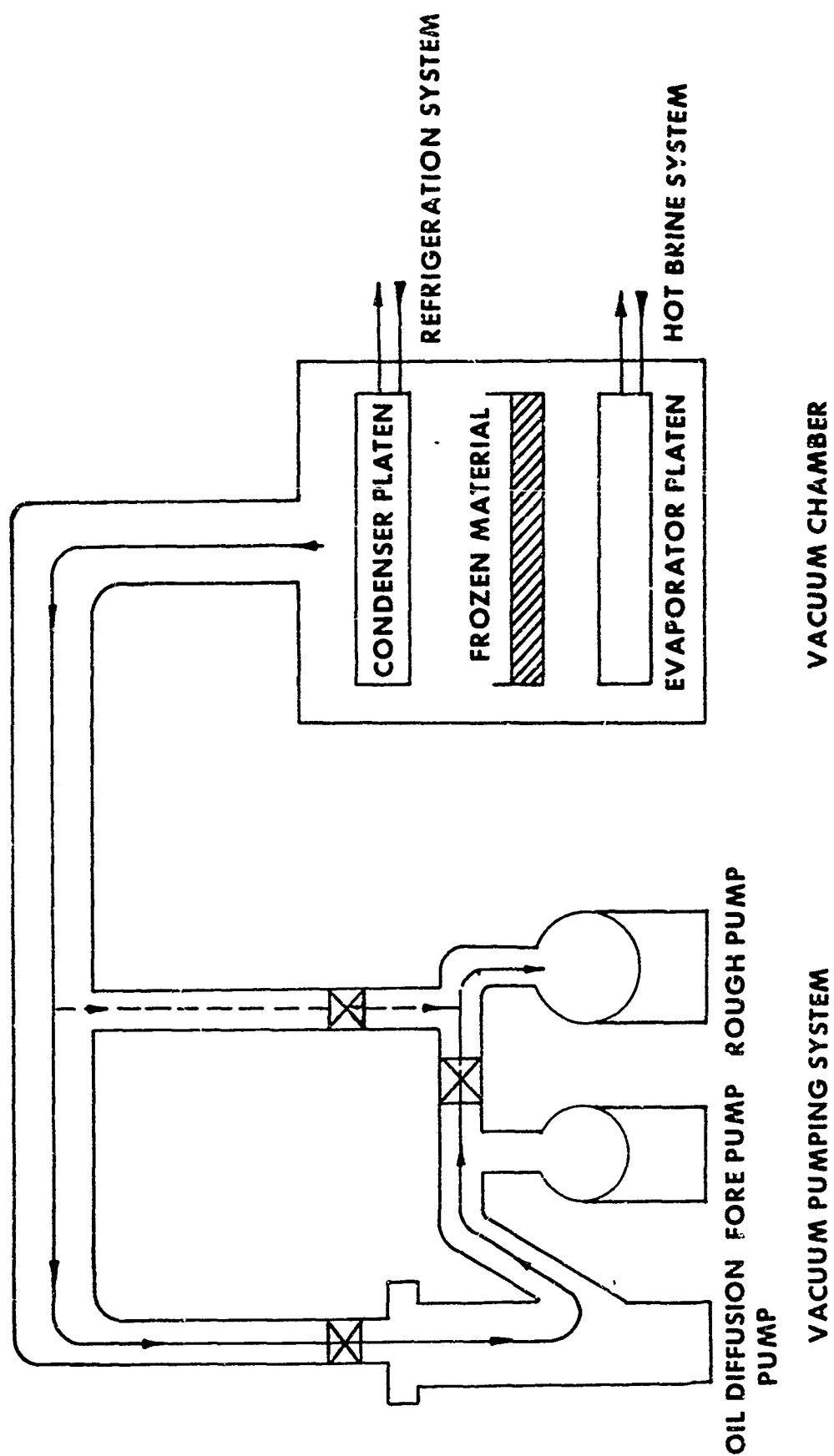


Figure 1. Equipment Arrangement in Typical Batch Freeze-Drying System.

II. THEORY OF HEAT AND MASS TRANSFER

A. GENERAL CONSIDERATIONS

The process of heat transfer in freeze-drying can be stated as follows. Heat must be transferred from the heater, through the evaporator shell, through the vapor space, through the frozen material to the interface where sublimation is actually taking place. Heat is also transferred from the desubliming vapor to the ice surface, through the ice cake and the condenser wall to the refrigerant, where it is absorbed and carried off.

The process of mass transfer begins at the subliming interface. The vapor formed there passes through the porous cake of previously dried material and into the vapor space separating the material and the condenser. From here it passes to the interface between the vapor space and the ice cake where it is desublimed and the process of mass transfer is ended.

It is these several steps of heat and mass transfer that actually govern the rates of drying.

1. Drying Stages

There are two stages in the drying process: the constant-rate period and the falling-rate period. The point which separates these two periods is termed the critical moisture content (CMC). Experimentally, this point is determined in the drying process when the temperature of the drying material begins to rise, because the constant-rate period is characterized by a constant temperature of the material. The CMC represents the point where all of the free moisture has been removed from the material.

Molochnaya^{2/} has proposed the following equation for the moisture content at this point:

$$w_c = \frac{A (t_m)^{0.5} (h)^{0.75}}{(P_m)^{0.2}} \quad (1)$$

where A = empirical constant (for gelatin, 6410)

2h = thickness of specimen being dried

P_m = pressure in system

t_m = final temperature of specimen

w_c = CMC

Lambert^{3/} gives this equation:

$$w_c = \frac{Eq}{\lambda_s} \quad (2)$$

where λ_s = heat of sublimation

$$Eq = (353) \cdot (\text{lb water sublimed}) / (\text{total watt-hr used})$$

During the constant-rate period, heat is supplied at a constant rate to the dryer. The faster heat can be supplied under the limitations of the heat and mass transfer rates, the faster the drying process. This heat is transferred through the cake and used to sublime the ice. All of this added heat is used in sublimation, allowing the material to remain at a constant temperature. Once the CMC is reached, the temperature of the material begins to rise because all of the free moisture has sublimed, resulting in less evaporative cooling. At this time, the heat to the dryer must be reduced to prevent overheating of the material and possible melting of some portions. The amount of heat reduction is dependent on the rates of heat and mass transfer. At the first evidence of temperature increase at any point of the material the heat should be reduced to avoid toasting at any one point, even though there may be free water remaining in other portions of the material.

Methods of heating include resistance wires, induction heating, high-frequency fields, dielectric heating, and infrared lamps.

In Flosdorf's book^{4/} Greaves points out that the control of the supply of latent heat of sublimation is the most important single factor in freeze-drying rates. How quickly heat can be applied will depend upon:

- (a) Capacity of refrigeration at condenser temperature reading;
- (b) Highest safe temperature at which material may be dried (for a given heat input, this temperature depends upon the degree of obstruction to vapor flow and upon condenser temperature);
- (c) Highest temperature at which it is safe to operate heaters; and
- (d) Rate of transfer of heat through frozen material.

The heat applied to the frozen material during drying produces a vapor pressure difference between the evaporating and condensing surfaces. This reflects a difference in temperature between the two surfaces.

Since rate of flow depends upon heat input (Clapeyron's equation), a formula may be written:

$$\text{Rate of flow} = \frac{\Delta P}{\text{obstructive resistance}} = (K) \cdot (\text{Watts}) \quad (3)$$

where ΔP = vapor pressure difference

K = volume constant in watt-hours per milliliter

Greaves^{5/} defines the unit of obstructive resistance as that which, under a vapor pressure difference of 0.01 mm Hg, passes vapor at the rate at which it is liberated by a heat input of one watt. Therefore:

$$\frac{P}{R} = \frac{W}{100} \quad (4)$$

where P = vapor pressure across the resistance in mm Hg

R = obstructive resistance

W = heat input in watts

2. Optimum Cycle

During the first stage of drying, heat must be introduced into the frozen material as rapidly as possible without causing it to soften or melt. At the same time, a maximum rate of flow of water vapor away from the evaporating surface must be established. To accomplish this rapid vapor flow, adequate passageways must be provided for the vapor, which must then be condensed or evacuated efficiently. However, supply of heat to frozen material is the controlling factor, according to Greaves.^{5/}

The basic limitations listed by Greaves^{5/} include:

(a) Heat must not be carried to the walls of the container which holds the product faster than the heat can be conducted through the frozen mass to a free surface where it is used to induce sublimation. Otherwise, melting will occur adjacent to the container wall.

(b) Heat can be carried down directly to the evaporating surface to avoid conductance through ice, but even here there is a limitation. As soon as sublimation has proceeded to an appreciable extent, the ice layer has receded from the level of the original surface so that the evaporating surface is confined within the interstices of the outer framework of porous dry solid. The heat must then be carried across this porous structure, but the temperature of this dry portion of the material must not be brought up to the level at which it is harmed.

Flosdorf^{4/} has determined that the difference between the partial pressure of water vapor that can be established within the vacuum system by the condenser and the vapor pressure of the evaporating material determines the theoretical maximum flow rate of vapor. The vapor pressure of the material at the temperature at which it should be dried determines the temperature at which the condenser should be maintained. Differential pressure between the partial pressure and the vapor pressure is the driving force.

For each of several temperatures there is a minimal temperature of condensing surface, below which there is very little further increase by further reduction in condenser temperature. To illustrate this point, x on each curve in Figure 2 represents the condenser (ice) surface temperature giving rise to maximum differential according to the Napier equation (laws of adiabatic gas flow through orifices), assuming no restrictive orifices in pipe lines, etc., other than in the interstices of the porous, dry, outer layer of material itself. This applies to the first stage of drying when ice is being sublimed.

Further, as drying proceeds, it is limited in rate by diffusion of vapor through the interstices of the porous dry outer layer of material, which act as orifices. By the Napier equation a differential between the vapor pressure at the surface of the condenser and the vapor pressure at the ice surface within the material, where the former is 55 per cent of the latter, will result in the maximum rate of flow obtainable. For example, this condition is satisfied by a material temperature of -18°C (for ice) and a condenser temperature of -25°C . The drying time is 21 hours.

During another run with a condenser temperature of -40°C , the drying time was the same but the jacket temperature of the drying chamber had to be raised from 60°C to 90°C in order to keep the material at -18°C . If the jacket temperature had not been raised, drying would have been retarded to 27 hours, with a -40°C condenser and the material at about -32°C . The pressure was 500 microns in both cases, but if the jacket had not been raised to 90°C with a -40°C condenser, this pressure would have had to be decreased to 225 microns. The efficiency of transfer of heat to the material is all-important in rapid drying, provided that the ice-condensing surface is below the critical level discussed above for obtaining the most rapid flow of vapor.

Reduction of the vapor pressure on the condensing surface to less than 55 per cent of the pressure on the drying side of each orifice, by laws of adiabatic gaseous flow, can produce no further increase in rate of flow through the orifice. In other words, beyond a certain point lowering the temperature of the condenser cannot compensate for small vapor lines or restrictions.

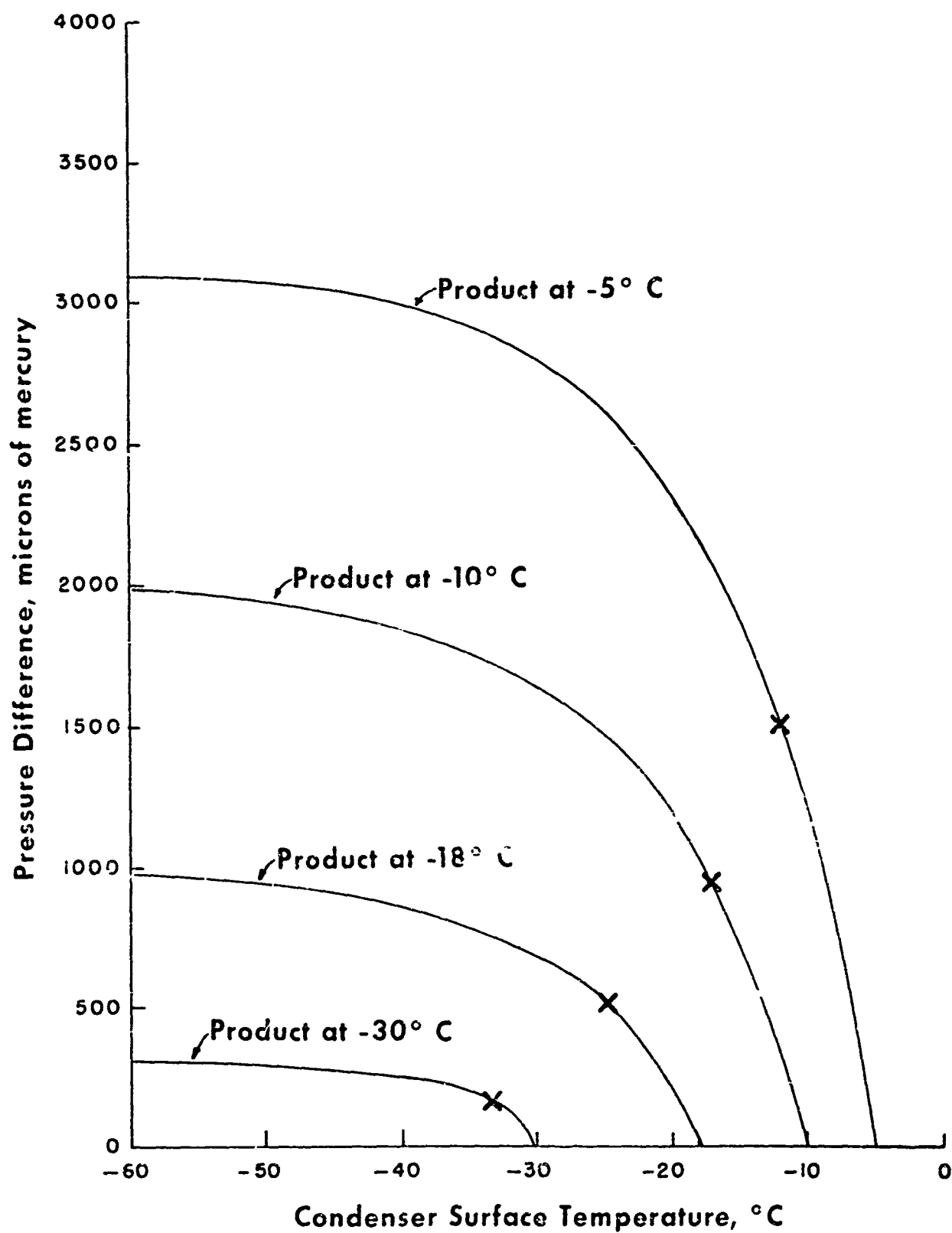


Figure 2. Driving Force in Terms of Differential in Pressure Produced by Various Condenser Temperatures for Products Maintained at Four Different Temperatures.

By use of condensers of great surface having constant efficiency and by a proper balance of all other factors concerned, a very important practical conclusion is obvious as a result of applying the law of flow. It becomes necessary to accept an ice surface with its poor heat transfer. However, by use of condensers of large surface area where the heat to be transferred per unit area is kept small, a higher surface temperature is adequate. Thus a higher temperature of refrigerant can be permitted. Condensers may be operated at temperatures much above any originally believed possible without sacrifice of effectiveness. With material at -21°C , a temperature of ice on the condenser of about -27°C produces a vapor pressure equal to 55 per cent of that of the material; thus a lower temperature at the ice surface is no advantage. Also, for greatest efficiency, the temperature of the material being dried must be no lower than necessary.

The final temperature to which the material may be taken in the second stage of drying and the vapor pressure of the product at that temperature also have a bearing on the required temperature for the condenser or pressures needed in direct pumping. This is related to the final residual moisture content obtainable. Whatever the vapor pressure of the dry product, the condenser or pump must establish a pressure that is lower.

In cases of hygroscopic products not stable at higher temperatures, a regenerable desiccant, either calcium sulfate or perchlorate, may be used advantageously to yield a low partial pressure of water vapor during the final stage. Since such a small actual weight of water is removed in this stage, regeneration of the desiccant is infrequent. This is a less expensive means than refrigeration to produce lower final condenser temperature or a steam ejector for lower total pressure.

B. SUBLIMATION

1. General Considerations

J.B. Lamberg presents a very complete review of sublimation in his thesis.^{3/} Almost all of the material that follows has been taken from this work. Equations generally apply to slabs, but with appropriate mathematical manipulation may be applied to any shape.

At the subliming interface, the absolute rate of sublimation, w_s^* , is:

$$w_s^* = \sigma P_s^* \sqrt{\frac{MH}{2\pi RT_s^*}} \quad (5)$$

where σ = accommodation coefficient

P_s^* = vapor pressure in equilibrium with T_s^*

T_s^* = temperature of subliming interface in degrees, abs.

MW = molecular weight of subliming substance

R = gas constant

The accommodation coefficient is used to compensate for the fact that some molecules collide with the interface and may be reflected instead of sublimed. Since the state of the gas does not affect the total rate of vaporization, Equation (5) is used to calculate evaporation rate under nonequilibrium conditions.

Previous investigations take the net sublimation rate, w , equal to the difference between evaporation and condensation rates at the interface, calculated by Equation (5) and assuming temperature equilibrium but not pressure equilibrium at the interface, therefore:

$$w = P_s^* \sqrt{\frac{MW}{2\pi RT_s^*}} - P_o \sqrt{\frac{MW}{2\pi RT_s^*}} = w_s^* \left(1 - \frac{P_o}{P_s^*}\right) \quad (6)$$

where P_o = initial vapor pressure of gas

The same accommodation coefficient is used for evaporation as for condensation.

Schrage^{6/} cites an error in this reasoning. Equation (5) for condensation assumes that molecular velocity distribution of gas in the vapor space is Maxwellian with respect to coordinates fixed in space. Since net mass transfer occurs, the system is not in equilibrium and ordinary Maxwell distribution does not apply. Schrage assumes that gas velocity distribution becomes Maxwellian with respect to the molecular stream velocity in a very few mean free paths from the interface. Therefore:

$$w_o = -\sigma P_o \sqrt{\frac{MW}{2\pi RT_o}} \Gamma = -\left(\frac{P_o}{P_s^*}\right) \left(\frac{T_s^*}{T_o}\right)^{1/2} \Gamma w_s^* \quad (7)$$

where Γ = correction factor to Equation (5), which is itself a function of net mass transfer rate.

T_o = gas temperature at interface, K^o

w_o = absolute condensation rate at interface

The factor Γ is defined by equations:

$$\phi = \frac{1}{2\pi^{1/2}} \left(\frac{W}{w_s^*} \right) \left(\frac{P_s^*}{P_o} \right) \left(\frac{T_o}{T_s^*} \right)^{-1/2} \quad (8)$$

$$\Gamma = e^{-\phi^2} - \phi \pi^{1/2} (1 - \operatorname{erfcy}) \quad (9)$$

where $\operatorname{erfcy} = \frac{1}{2\pi^{1/2}} \int_0^x e^{-x^2} dx$, the error integral.

Addition of total evaporation and condensation rates from Equations (5) and (7) yields net transport:

$$w = w_s^* \left[1 - \left(\frac{P_o}{P_s^*} \right) \left(\frac{T_s^*}{T_o} \right)^{1/2} \Gamma \right] \quad (10)$$

The factor Γ is uniquely related to w/w_s^* for the special case where T_o/T_s^* is equal to one. If P_o/P_s^* is eliminated from Equations (9) and (10), ϕ may be plotted as a function of w/w_s^* . Values of Γ are then calculated from various w/w_s^* values by using Equation (9) with the plot shown in Figure 3. In applying Schrage's^{6/} theory to the case of $T_o/T_s^* = 1$ for net evaporation, the ratio P_o/P_s^* decreases with w/w_s^* ; it increases for net condensation. Schrage's modified theory predicts these critical or maximum mass transfer ratios:

$$w/w_s^* = 0.77 \text{ for evaporation}$$

$$w/w_s^* = 13 \text{ for condensation}$$

The above theories should be applied to mass transfer processes not far from equilibrium. The accommodation coefficient for ice-water has been found to be one, assuming no temperature dependency.

Kramers and Stemerding^{7/} give equivalent equations for sublimation rates. On the basis of Knudsen's formula, they propose a theoretical sublimation rate, w_o :

$$w_o = \pi_{es} \sqrt{\frac{M}{2\pi R T_{es}}} \quad (11)$$

where M = molecular weight of evaporating substance

π_{es} = vapor pressure of evaporating substance

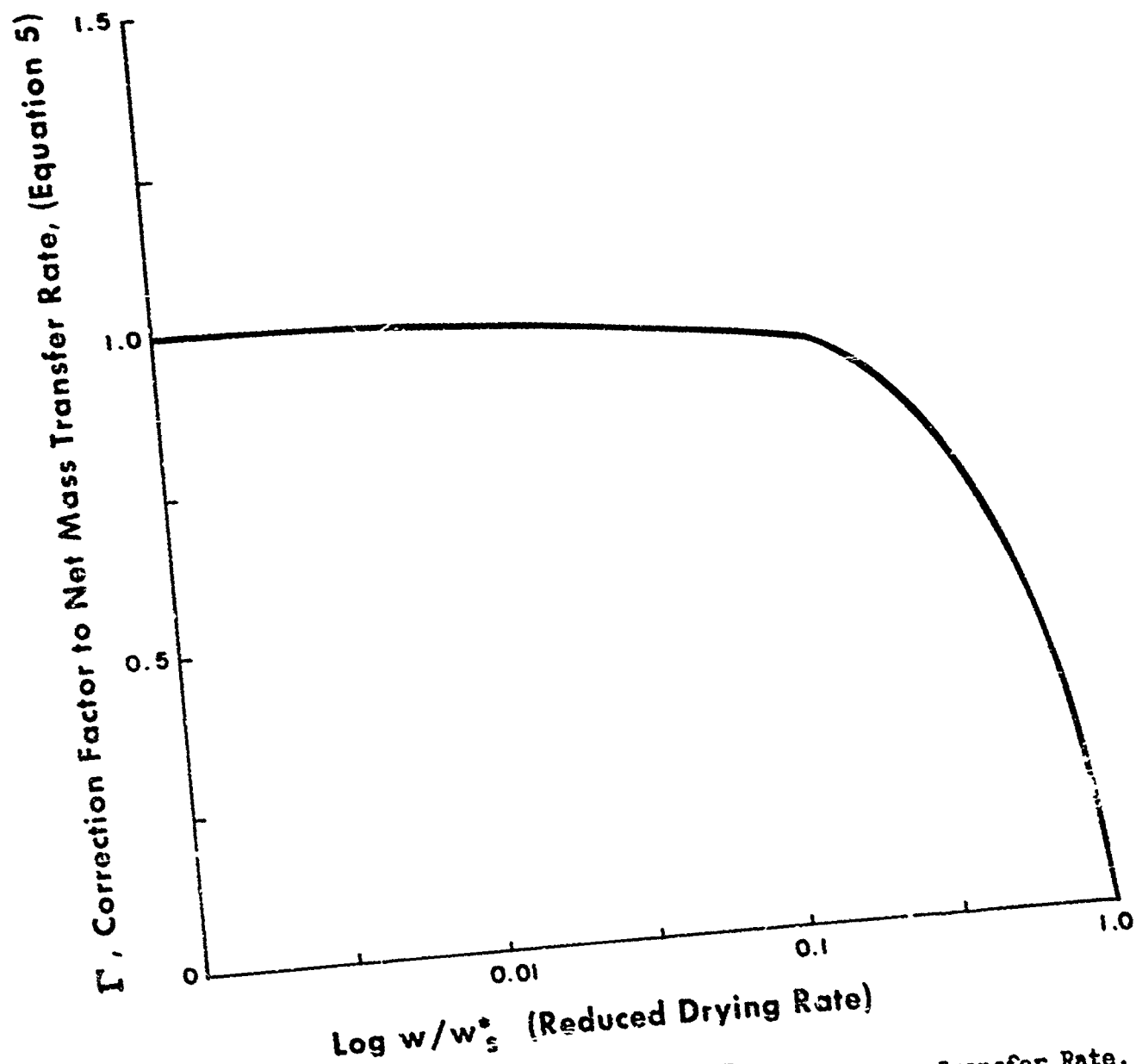


Figure 3. Relation of Correction Factor, Γ , to Net Mass Transfer Rate, W , where T_0/T_s^* Equals One.

R = gas constant

T_{es} = absolute temperature of surface

This assumes an accommodation coefficient of unity, which implies no rebounding of any of the molecules on collision.

In freeze-drying practice a similar formula is used:

$$w = f \pi_{es} \sqrt{\frac{M}{2\pi R T_{es}}} = f w_0 \quad (12)$$

where f , the coefficient of evaporation, is between zero and one.

In this formula the following effects causing w to be less than w_0 may have been collected:

(a) Accommodation coefficient is less than one.

(b) Pressure in vapor space is not zero, so that vapor molecules have chance of returning to the evaporator surface. This pressure will be at least the vapor pressure (π_{cs}) of the condenser surface, which is eventually increased as a result of resistance to vapor flow from the evaporator to the condenser either by diffusion through noncondensable gas (air) or by wall friction. The latter effect is important in freeze drying because the layer of dry product obstructs this flow.

(c) σ , the accommodation coefficient, may be limited by the rate at which heat is being supplied to the evaporator and by the rate of withdrawal of the vapor by condensation, absorption, or pumping:

$$\sigma = 0.94 \pm 0.06 \text{ for ice } -60^\circ\text{C} \longrightarrow -85^\circ\text{C}$$

If there is no frictional resistance between evaporator and condenser, and if the total water vapor plus air pressure in the system is $P = (P + \pi')$, the rate of sublimation is given by:

$$w = w_0 \left(1 - \frac{\pi'}{\pi_{es}} \right) = \frac{MD}{RT} \ln \left(\frac{P - \pi_{cs}}{P - \pi'} \right) \quad (13)$$

where M = molecular weight of evaporating surface

R = gas constant

D = coefficient of diffusion

P = total pressure

π' = partial pressure of water vapor

\bar{T} = average absolute temperature

l = distance from evaporator surface to condenser surface

π_{es} = partial pressure of evaporating surface

π_{cs} = partial pressure of condensing surface

The theory is that two resistances exist, in series, to the transfer of vapor. The term $w_0 (1 - \frac{\pi'}{\pi_{es}})$ gives the net transport by free evaporation between the evaporator surface and the adjacent vapor, which has partial water vapor pressure π' ($\pi_{es} \leq \pi' \leq \pi_{cs}$). The right-hand expression represents the diffusional flow of water vapor through air from π' near the evaporator to π_{cs} at the condenser surface, over distance, l , between them.

2. Special Cases

1. If the air pressure at the condenser is relatively low, or $P \approx \pi_{cs}$, then also $\pi' \approx P$, giving

$$w = w_0 \left(1 - \frac{\pi_{cs}}{\pi_{es}} \right) \quad (14)$$

2. If w is much less than w_0 , e.g., by an insufficient supply of heat, it follows from Equation (13) that $\pi' \approx \pi_{es}$. In the absence of air this means that $\pi_{es} \approx \pi_{cs}$ i.e., the surface temperatures of evaporator and condenser are equal. In the presence of air the equation becomes:

$$w = \frac{MDP}{RTl} \ln \left(\frac{P - \pi_{cs}}{P - \pi_{es}} \right) \quad (15)$$

where the symbols are the same as those in Equation (13).

3. Application

For the application of the theory, a McLeod vacuum gauge gives the total pressure, $P^*_{H_2O} + P^*_{air}$, and the ice surface temperatures t_{es} and t_{cs} have to be known. They can be derived from the measured temperatures t_e and t_c by correcting for the temperature drop over the layer of air. The latter quantity can be calculated from the heat flow corresponding

to the measured rate of sublimation and the thickness and thermal conductivity of the ice layers. In experiments carried out for t_e between -20°C and -25°C and t_c between -30°C and -37°C at an air pressure less than one micron of mercury, the rate was of the order 10^{-4} gm/cm²sec; i.e., two to four per cent of the maximum rate at these temperatures. Consequently, in these runs t_{es} is approximately equal to t_{cs} . From the measured temperature differences, $t_e - t_c$, and the corresponding rates of evaporation and ice thickness, the thermal conductivity was found to range between 5.0×10^{-3} and 6.1×10^{-3} calories per centimeter degree Centigrade, giving an average value of $5.7 \pm 0.4 \times 10^{-3}$ at a mean temperature of -25°C . The agreement with the generally accepted value of 5.7×10^{-3} at -20°C is satisfactory.

The following situation demonstrates the existence of a maximum rate of evaporation. Ice is evaporated at a certain constant rate with a condenser temperature of about -30°C and lowest possible residual air pressure, so that $t_{es} = t_{cs}$. If the condenser temperature is lowered and the sublimation rate remains the same, a difference between t_{es} and t_{cs} begins to appear. It increases in such a way that finally t_{es} will become constant for very low condenser temperature. This is shown in Figure 4.

It is evident that at the corresponding temperatures t_{es} and t_{cs} , these are the maximum rates which can be compared with the equation $w = w_0 \left(1 - \frac{\pi_{cs}}{\pi_{es}} \right)$. As an example, the lowest curve has a point $t_{es} = -57^\circ\text{C}$, $t_{cs} = -64^\circ\text{C}$. With these values, w gives a rate of 1.3×10^{-4} gm/cm²sec, whereas the measured rate was 1.2×10^{-4} . Agreement is good, showing an accommodation coefficient of nearly 1.0.

Table I presents data for air pressures less than one micron of mercury.

The values of unity for $w_{\text{expt}}/w_{\text{calc}}$ support the above theory. This is especially important, because in most experiments the mean free path of the molecules was less than the distances, l , whereas Equation (13) is generally accepted only for cases in which the mean free path greatly exceeds l . Since Equation (13) has been derived from an accommodation coefficient of unity, these results show that this assumption is very probably correct in the temperature range from -40 to -60°C .

Only at low condenser temperatures and at relatively great distances between the evaporator and condenser are rather small $w_{\text{expt}}/w_{\text{calc}}$ ratios found. Probably, wall friction and, to a minor extent, the pressure of the residual air are responsible.

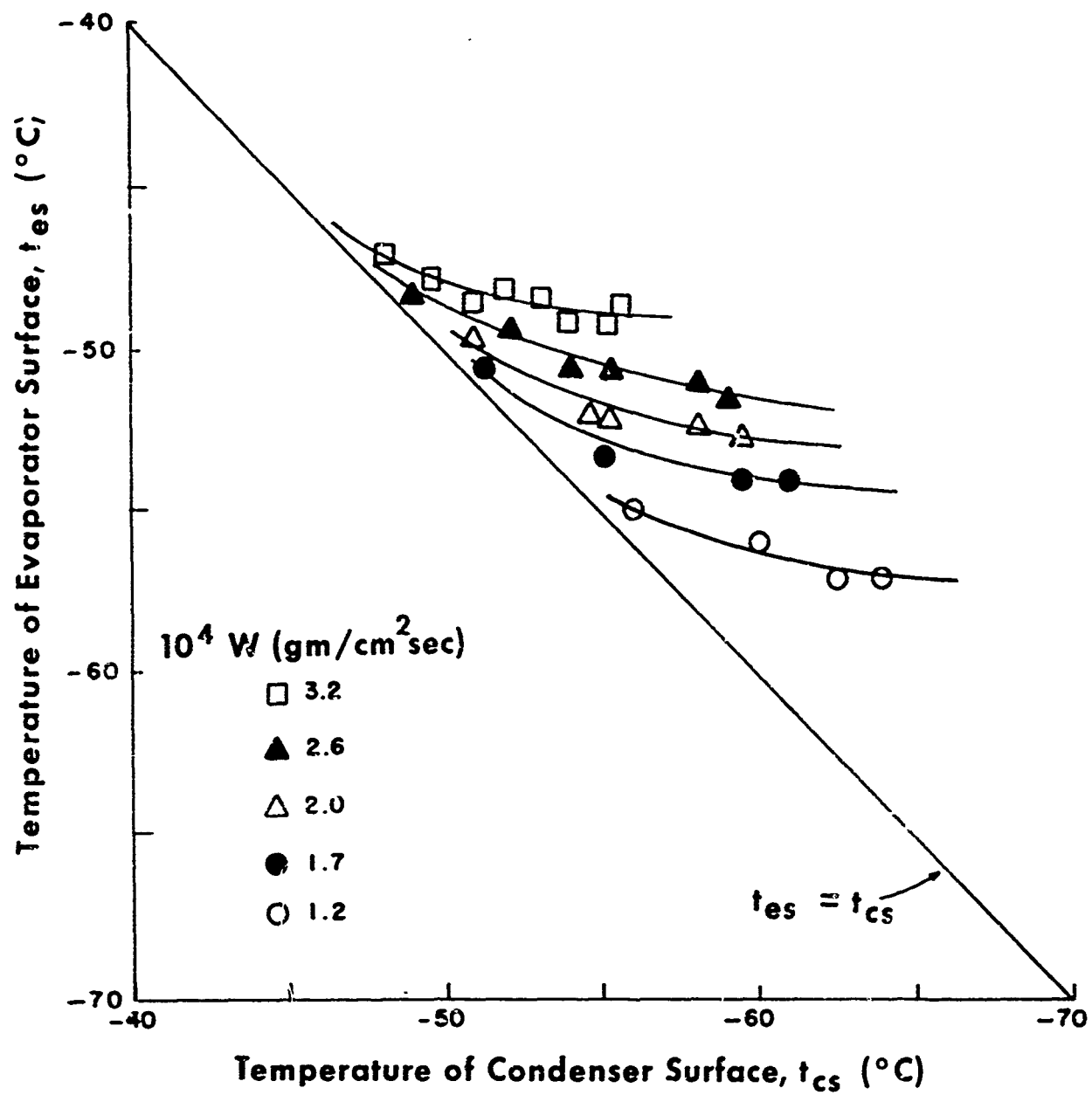


Figure 4. Sublimation Rates as a Function of Evaporator and Condenser Surface Temperatures.

TABLE I. SUBLIMATION RATES FOR ICE AT PRESSURES LESS THAN ONE MICRON OF MERCURY

l^a/mm	$t_{es}, ^\circ\text{C}$	$t_{cs}, ^\circ\text{C}$	$t_{es-cs}, ^\circ\text{C}$	Calculated ^{b/} Sublimation Rate, 10^4 $\text{gm}/\text{cm}^2\text{sec}$	Experimental Sublimation Rate, 10^4 $\text{gm}/\text{cm}^2\text{sec}$	$\frac{w \text{ expt.}}{w \text{ calc.}}$
12	-44.0	-47.9	3.9	3.6	5.21	1.4 ± 0.2
	-49.2	-57.0	7.8	3.6	3.54	1.0 ± 0.15
	-52.6	-61.8	9.2	2.4	2.09	0.9 ± 0.15
	-54.8	-65.5	9.3	2.0	1.55	0.8 ± 0.20
	-57.0	-64.9	7.9	1.5	1.16	0.8 ± 0.15
	-58.8	-64.4	5.6	0.80	0.59	0.7 ± 0.15
	-61.7	-70.0	8.3	0.75	0.675	0.9 ± 0.05
20	-41.3	-47.6	6.3	7.0	6.60	0.9 ± 0.05
	-46.2	-52.7	6.5	4.4	5.00	1.1 ± 0.1
	-46.0	-54.6	8.6	5.1	3.72	0.7 ± 0.1
	-50.4	-58.9	8.5	3.1	3.31	1.1 ± 0.1
	-50.4	-58.6	8.2	3.0	2.78	0.9 ± 0.1
	-52.2	-62.2	10.0	2.6	2.23	0.9 ± 0.1
	-54.3	-64.2	9.9	2.0	1.60	0.8 ± 0.05
	-56.4	-66.7	10.3	1.6	1.15	0.7 ± 0.05
44	-61.3	-70.1	8.8	1.0	0.607	0.6 ± 0.05
	-43.3	-47.6	4.3	4.3	2.70	0.6 ± 0.05
	-48.3	-60.8	12.5	4.5	2.30	0.5 ± 0.1
	-49.0	-59.4	10.4	4.0	2.65	0.7 ± 0.1
	-51.2	-61.7	10.5	3.1	1.86	0.6 ± 0.1

a. Distance between evaporator and condenser.

b. Calculated by Equation (13).

In a number of runs with constant evaporation rate and condenser temperature, the air pressure was increased in a few steps from 10^{-3} to 0.4 millimeters of mercury. This caused the evaporator temperature to increase correspondingly. From temperature and pressure measurements, P and π_{cs} were derived; π' could be calculated from Equation (14): $w = w_0 \left(1 - \frac{\pi'}{\pi_{es}}\right)$.

Only for high air pressures was the difference between π_{es} and π' insignificant. Thus, from each run, two to six values of $(P - \pi') / (P - \pi_{cs})$ were obtained.

It will be seen from the following equation that, for a constant sublimation rate and average absolute temperature \bar{T} , this form has to be independent of pressure. So for each run the average value $(P - \pi') / (P - \pi_{cs})$ was calculated as shown in Table II.

TABLE II. SUBLIMATION RATES FOR ICE WHERE PRESSURE INCREASED FROM 10^{-3} TO 0.4 MILLIMETERS OF MERCURY

l, mm	$w \text{ expt}, 10^4$ $\text{gm/cm}^2\text{sec}$	$t_{cs}, ^\circ\text{C}$	\bar{T}	$\frac{RT_0^{3/2}}{D_0 P_0 M}$	$\frac{1 \bar{w}}{\bar{T}^{1/2}} \left(\frac{P - \pi'}{P - \pi_{cs}} \right)$	$\frac{a}{\text{calc}}$	$\left(\frac{P - \pi'}{P - \pi_{cs}} \right)_{\text{expt}}$
20	0.48	-20	254	0.41	0.66		0.68 ± 0.03
	0.48	-35	241	0.42	0.66		0.65 ± 0.02
	0.60	-50	229	0.54	0.58		0.56 ± 0.02
	0.76	-39	238	0.67	0.51		0.60 ± 0.08
	0.84	-37	243	0.74	0.48		0.49 ± 0.08
	0.95	-36	240	0.84	0.43		0.48 ± 0.08
	1.06	-48	235	0.94	0.39		0.29 ± 0.04
	1.42	-36	241	1.25	0.29		0.49 ± 0.10
	1.68	-44	236	1.49	0.23		0.22 ± 0.06
44	0.57	-46	233	1.01	0.36		0.40 ± 0.06
	1.05	-47	235	1.98	0.14		0.15 ± 0.05
	2.05	-44	238	3.60	0.03		0.17 ± 0.05

a. Calculated by Equation (17).

For comparison with

$$w = \frac{MDP}{RT\bar{T}} \ln \left(\frac{P - \pi_{cs}}{P - \pi'} \right) \quad (15a)$$

use has been made of the approximate proportionality between the diffusion coefficient D and $T^{3/2}/P$, thus

$$\frac{DP}{\bar{T}} = \frac{D_0 P_0}{T_0^{3/2}} \sqrt{\bar{T}} \quad (16)$$

Subscript refers to values at 0°C and one atmosphere. Substituting Equation (16) becomes:

$$\left(\frac{P - \pi'}{P - \pi_{cs}} \right) = \text{expt} \left(\frac{-RT_0^{3/2}}{D_0 P_0 M} \frac{1 w}{\sqrt{\bar{T}}} \right) \quad (17)$$

In Equation (17) w is the mass flow of water vapor per unit surface of the diffusion space.

In the experimental arrangement, the surface available for diffusion was circular, 75 millimeters (mm) in diameter, a distance of about six mm from the evaporator surface. For the rest of the distance, l , a diameter of 96 mm was available. From this, the average flow density \bar{w} can be calculated to be $0.73 w_{\text{expt}}$ for $l = 20$ mm and $0.66 w_{\text{expt}}$ for $l = 44$ mm. These values were used in the computations for Table II.

Bradish^{8,9} quotes Knudsen that the maximum rate of vaporization from an ice surface at absolute temperature T is expressed by the equation:

$$G_{\text{max}} = 0.244 \sigma \frac{P_t}{\sqrt{T}} \text{ gm/cm}^2\text{sec} \quad (18)$$

where P_t = equilibrium vapor pressure of ice in mm Hg (sat.)

0.244 = function of molecular weight of vaporizing material

σ = condensation coefficient, which expresses fraction of vapor molecules entering mass of ice, following collision with its surface

T = absolute temperature

Bradish accepts σ equal to one. Implication of σ equal to one is that every molecule colliding with the ice surface is immediately condensed, although it may be evaporated later. Thus, every molecule leaving the evaporating surfaces will suffer collisions in the partially dry material and interspace and will eventually collide with and condense on the surface of the frozen material or the condenser ice. Effective rate of sublimation is not equal to the rate of vaporization from the surface of the frozen material, but to the mass of water vapor actually transferred in unit time from frozen material to condenser ice.

C. HEAT TRANSFER IN FREEZE DRYING

Lambert³ gives the following equation for conductive heat transfer in freeze drying.

$$w = \frac{t_v^* - t_s}{\lambda_s \left[\frac{1}{h} + \frac{B}{K_b} + \frac{X}{K_i} + R_i \right]} \quad (19)$$

where w = drying rate per unit area

h = heat transfer coefficient of heating medium

B = thickness of containing wall

X = distance from containing wall to subliming interface

R_i = resistance at container ice boundary

t_v^* = temperature of heating medium

k_b = thermal conductivity of container

t_s = temperature of subliming surface

λ_s = heat of sublimation

k_i = thermal conductivity at container ice boundary

As the ice is completely sublimed the temperature of the product rises. According to Abelow and Flosdorf,¹⁰ this represents maximum dehydration as far as is possible at this stage of the cycle. When the product reaches a properly chosen control temperature the heat input is reduced by allowing the platen temperature to drop. The product temperature is allowed to rise to a predetermined balancing temperature equal to the final temperature of the platen. The product is retained at this temperature over the so-called balancing-out period.

Abelow and Flosdorf¹⁰ concluded that, compared with the flat-tray design, ribbed-tray design with a given loading yields greater output at any usable temperature condition tested. In addition, ribbed design, compared with flat design, shows more rapid increase in output as heating intensity is increased. To compensate for volume occupied by the ribs, the layer of product should be thicker to maintain comparable tray loading based on drying platen area. Actual metallic tray heating surface to which pellets were exposed in ribbed trays is approximately 215 per cent greater than in flat trays.

The 100° to 40°C heat range with black ribbed trays produces the best output with acceptable final content of residual moisture.

In order to calculate the rate of drying during the constant-rate period, either the mass transfer equation or the heat transfer equation may be used:

$$\text{Mass Transfer: } w = k'y (H_i - H)A$$

$$\text{Heat Transfer: } w = \frac{hy (t - t_i)A}{\lambda_i} \quad (20)$$

where w = rate of evaporation, pounds per hour

A = drying area, square feet

h_y = heat transfer coefficient, Btu/(ft²) (hr) (°F)

k'_y = mass transfer coefficient, lb/(ft²) (hr) (unit humidity difference)

H_i = humidity of air at interface, pounds H₂O/pounds dry air

H = humidity of air, pounds H₂O per pounds dry air

t = temperature of air, °F

t_i = temperature at interface, °F

λ_i = latent heat at temperature, t_i , Btu per pound

When the air is flowing parallel with the surface of the solid, the heat transfer coefficient may be estimated by the dimensional equation:

$$h_y = 0.128 G^{0.8}$$

where G is the mass velocity in lb/(ft²) (hr)

When the flow is perpendicular to the surface, the equation is:

$$h_y = 0.37 G^{0.37}$$

D. INTERPHASE MASS TRANSFER

From Schrage,^{6/} at the evaporator:

$$w/w_{s^*} = \left[1 - \left(\frac{P_o}{P_{s^*}} \right) \left(\frac{T_{s^*}}{T_o} \right)^{1/2} \right] \Gamma_s \quad (21)$$

$$w = \sigma \beta_s \left[P_{s^*} - P_o \left(\frac{T_{s^*}}{T_c} \right)^{1/2} \right] \Gamma_s \quad (21a)$$

$$\text{where } \beta_s = \frac{w_{s^*}}{\sigma P_{s^*}} = \sqrt{\frac{MW}{2 \pi RT_{s^*}}} \frac{\text{Pounds}}{(\text{mmHg}) (\text{ft}^2) (\text{hr})}$$

For water, $\beta_s = \frac{2447.5}{(T_s)(^{\circ}R)}$ in pounds per millimeter of mercury-square foot-hour

For the condenser:

$$\frac{w}{w_s^*} = \left[\left(\frac{P_c}{P_c^*} \right) \left(\frac{T_c^*}{T_c} \right)^{1/2} \right] \Gamma_c^{-1} \quad (22)$$

$$w = \sigma \beta_c \left[P_c \left(\frac{T_c^*}{T_c} \right)^{1/2} \right] \Gamma_c - P_c^* \quad (22a)$$

T_c^*, P_c^* = temperature and equilibrium vapor pressure of ice, at condenser interface

P_c, T_c = true interfacial values in gas phase.

Accurate Approximation:

$$\sigma = 1$$

$$\left(\frac{T_c^*}{T_c} \right)^{1/2} = 1$$

E. MASS TRANSFER THROUGH DRIED MATERIAL

Movement of water through a partially dry cake of material above the subliming interface is attributable to one of the following:

- (a) Bulk flow under a pressure gradient
- (b) Diffusion of water vapor in gas phase
- (c) Liquid diffusion through solid

During the constant-rate period, drying usually occurs from the receding ice zone; during the falling-rate period drying proceeds through the dried cake.

The actual structure of the dried layer is affected by solids content, type of material being dried, rate of freezing, solids redistribution during freezing, and shrinkage due to thawing during drying.

Strickland-Constable and Bruce^{11/} developed the following equation for the flow water vapor through freeze-dried clay:

$$w = \frac{(\Delta P)^2}{R_m L} \quad (23)$$

where w = drying rate

L = thickness of dry layer

R_m = specific resistance of the material being dried

ΔP = pressure drop through the cake.

As the downstream pressure is negligible, ΔP is equal to the equilibrium vapor pressure at the temperature of the subliming surface. The above relation has doubtful application.

Stephenson^{12/} and Carman^{13/} combined classical diffusion and kinetic theories to characterize mass transfer in vacuum distillation or sublimation. Three operating regions were identified by ratios of equilibrium vapor pressure at subliming interface, P_s^* , to total pressure, \bar{P} ,

Region	P_s^*/\bar{P}	Rate-Controlling Step
1	<1.0	diffusion between evaporator and condenser
2	>1.0	molecular sublimation, interphase mass transfer controlling
3	$\cong 1.0$	transition region, neither process controlling

Linear velocities of water vapor may be sonic in pores of dried material above subliming surface, if the resistance to vapor transport in these pores is high. Thus, when the critical pressure ratio for water vapor of 0.55 is reached, a further decrease in the downstream (space between evaporator and condenser) pressure has no effect on drying rate or subliming surface temperature. This effect has been observed by Flosdorf.^{14/}

The Grashof Number (N_{Gr}) was also used for correlation of heat and mass transfer data:^{15/}

$$N_{Gr} \left] h, m \right] = \left[\frac{gL^3}{U^2} \left(1 - \frac{M_o T_c}{M_c T_o} \right) \right] \quad (24)$$

where g = acceleration of gravity

L = characteristic length of system

U = kinematic viscosity

M = molecular weight

T = temperature

subscripts:

o = evaporator

c = condenser

h = heat

m = mass

Jacob indicates that with heat transfer N_{Gr} as low as 1600, free convectional contribution to heat transfer in an enclosed horizontal air layer is negligible. Therefore, it seems unlikely that free convection mass transfer is a significant factor in freeze drying, as N_{Gr} is almost always below 1100.

Guthrie and Wakerling^{16/} show that turbulent flow is rarely a consideration in vacuum work below 2 mm of Hg. The same authors give the following criteria which define the free molecular and viscous regions in terms of the Knudsen number:

Free molecular flow (turbulent) $N_{Kn} > 0.33$

Transitional region $10^{-2} < N_{Kn} < 0.33$

Laminar flow (viscous) $N_{Kn} < 10^{-2}$

where $N_{Kn} = \bar{l}/L$

\bar{l} = mean free path

L = shortest dimension characteristic of flow channel

$\bar{l} \approx T/P$

In commercial freeze dryers the flow from the evaporator to the condenser is almost always laminar. Flow through partially dry solid above the subliming surface is usually free molecular or transitional.

Bulk flow under a pressure gradient through a porous cake may be free molecular, transitional, or laminar; although free path considerations indicate that free molecular usually predominates. For free molecular flow, collisions between gas and solid molecules are the most frequent, and flow of water vapor is independent of air pressure. Drying probably occurs by process of successive evaporation and condensation.

Assuming drying takes place only at the free ice surface, Guthrie and Wakerling^{16/} present a relation for free molecular flow through a channel connecting two regions of different pressure derived from the theory of Equation (5). This expression is adaptable to the flow of water vapor through a porous dry layer if a suitable dimensionless constant (correction factor) K_{fm} , dependent on flow geometry and void fraction of material being dried, is added. It is further assumed that partial pressure of H_2O varies continuously with cake thickness:

$$w_{bf} = K_{fm} \beta_s d \frac{\partial p}{\partial x} \quad (25)$$

w_{bf} = bulk flow rate, mass/time-area

d = average diameter of flow channel (ice crystal diameter)

β_s = defined in Equation (21a)

For the transitional flow region, Knudsen^{17/} has derived the formula for pressure drop in long circular pipes. This formula is analogous to Equation (25) except for a multiplying factor, $J(N_{Kn})$. Guthrie and Wakerling^{16/} have tabulated this factor for various Knudsen numbers. By introducing this term, Equation (25) may be applied to the present case.

Intermolecular gas phase collisions are most common in the viscous or laminar region, and flow rate is not independent of the air pressure. The starting point for most investigations of flow through porous media is Darcy's Law:

$$w_{bf} = - \frac{\psi \bar{\rho}_0}{\mu} \frac{d\bar{T}}{dx} \approx - \frac{\psi \bar{\rho}_0}{\mu} \left(\frac{dp}{dx} \right) \quad (26)$$

where ψ = cake permeability in units of (length)²

$\bar{\rho}_0$ = average gas density in the cake

μ = gas viscosity

The right-hand side of this equation is approximately correct, since the partial pressure of the air in the cake is usually small compared with the partial pressure of the water vapor. Generally speaking, the permeability is proportional to the square of the effective flow channel diameter, d , and a constant, K_L , dependent on flow geometry and fraction voids.

The diffusion of water vapor in the gas phase occurs only when N_{Kn} is in the laminar flow region:

$$w_d = -\epsilon^2 D_v \left(\frac{\partial \rho_w}{\partial x} \right) \quad (27)$$

w_d = rate of diffusional mass transfer

D_v = diffusion coefficient for water vapor in air

ρ_w = water vapor density = PM/RT

ϵ = void fraction

Equations (26) and (27) give the total mass flux of water vapor at any position in the dry cake for laminar flow.

During constant-rate periods, the partial pressure gradient may be assumed independent of time (steady state). Integration of the individual flow equations between L and x , the boundaries of the dry layer, yields the net flow rate if no change in flow mechanism occurs between the limits:

$$\text{Flow, molecular: } w = \frac{P_o - P_i}{L - x} \left(K_{fm} \beta_s d \right) \quad (28)$$

$$\text{Viscous : } w = \frac{P_o - P_i}{L - x} \left(\frac{D_v MW}{RT} \epsilon^2 + \frac{\psi P_o}{H} \right) \quad (28a)$$

$$\text{Transitional : } w = \frac{P_o - P_i}{L - x} \left[K_{fm} \beta_s d J (N_{Kn}) \right]$$

where P_o and P_i are the partial pressure of water at the free ice and dry cake surfaces, respectively.

After the free ice is removed, liquid diffusion mechanism must be considered and nonisothermal unsteady state conditions are encountered throughout the porous bed. The rate of flow per unit area by liquid diffusion, w_D , is:

$$w_D = -D_L \frac{\partial w}{\partial x} (1 - \epsilon)^2 \delta \quad (29)$$

where D_L = diffusion coefficient for water through the solid

w = moisture content per unit mass of dry solid at x .

δ = bulk density of dry solid

ϵ = void fraction

The term containing the void fraction accounts for the fact that the area for liquid diffusion is less than the cake cross-sectional area. Addition of Equation (29) to the corresponding equation for vapor diffusion or bulk flow gives the total rate of moisture flow at x . Thus, for free molecular bulk flow, total rate of moisture movement, w_x , is:

$$w_x = -K_{fm} d \left(\frac{\partial P}{\partial x} \right) - (1 - \epsilon)^2 D_L \left(\frac{\partial w}{\partial x} \right) \delta \quad (30)$$

A water balance on thickness element, dx , of the slab is obtained by equating the difference between the input and the output of water to the change in moisture content within dx . The difference between the input and output water is found by differentiating Equation (30) with respect to x and multiplying by dx .

$$w_{in} - w_{out} = \frac{\partial w_x}{\partial x} dx \quad (30a)$$

Water content of the element is made up of the sum of the liquid and vapor portions:

- (a) Liquid water in the solid/unit cake area is equal to $w \delta dx$
- (b) Water vapor in the pores of the solid/unit area is equal to

$$\rho_w \epsilon dx = \frac{PMW \epsilon dx}{RT}$$

The change in the moisture content of the element dx per unit cake area is obtained by differentiating the total water content with respect to time. The latter is then equated to the input-output of water:

$$\beta \frac{\partial}{\partial x} \left[-K_{fm} \beta_s d \left(\frac{\partial P}{\partial x} \right) - \delta (1 - \epsilon)^2 D_L \left(\frac{\partial w}{\partial x} \right) \right] = - \frac{\partial}{\partial \theta} (w \delta + P_w \epsilon) \quad (31)$$

Equation (31) is a general equation describing the mass transfer in the falling-rate period of freeze drying. It may be simplified if the values of K_{fm} , β_s , d , ϵ , D_L , and δ are assumed constant with x .

$$K_{fm} d \left(\frac{\partial^2 P}{\partial x^2} \right) + (1 - \epsilon)^2 \delta D_L \left(\frac{\partial^2 w}{\partial x^2} \right) = \delta \left(\frac{\partial w}{\partial \theta} \right) + \epsilon \left(\frac{\partial P_w}{\partial \theta} \right) \quad (32)$$

Graphical solutions to similar expressions have been obtained for several boundary conditions.

F. MASS TRANSFER THROUGH THE VAPOR SPACE

Bradish^{8/} states that the retardation effect of the porous dry mass above the receding ice surface of the material being dried is small. He believes that the transfer of vapor is limited by the rate of diffusion.

Bradish^{8/} theorizes that the interspace gap is so short that molecules are transferred almost without collision, then the sublimation rate is equal to the rate of vaporization at the surface of frozen material minus the rate of vaporization at the surface of the condenser ice.

If numerous collisions occur in the interspace, this simple difference equation no longer applies. It must include terms expressing proportion of molecules which traverse the interspace, as distinct from those which eventually condense on the surface from which they were evaporated.

In drying systems in which evaporation and condensing surfaces are widely separated, or in which the permanent gas partial pressure is high, the sublimation rate is low as a consequence of the numerous molecular collisions in the interspace.

Assuming parallel condensing and evaporating surfaces, the flow channel between the sample and the condenser is a rectangular duct. If free molecular flow pertains, the rate of water vapor transport, w , is:

$$w = 8/3 K \left[\frac{ab}{a+b} \right] \left(\frac{P_i - P_c}{Z} \right) \quad (33)$$

where a, b = dimensions of rectangular cross section

Z = distance from condenser to cake

K = constant, dependent on ratio a/b (tabulated by Guthrie and Wakerling when a/b is $1/2$, K is 1.151)

In the viscous region (N_{Kn} less than 0.01), the vapor movement is controlled either by vapor diffusion through stagnant air or by diffusion and bulk flow under a pressure gradient. Stefan's diffusion equation described the former case:

$$w = D_v \frac{\bar{M} W (P_i - P_c)}{RT Z p_{1m}} \quad (34)$$

D_v = diffusion coefficient for water vapor through air

MW = molecular weight of water

P_{1m} = log mean air pressure = $p_c - p_i / \ln p_c / p_i$

\bar{P} = total pressure ($p_c + p_i$)

z = distance from cake surface to condenser

P refers to water vapor, p to air

For ideal gases at constant T and total P , a general diffusion equation (neglecting thermal, pressure and forced diffusion) is:

$$w = \frac{P}{\bar{P}} \left[w + \frac{M_H}{M_a} w_a \right] - \frac{D_H M_H}{RT} \frac{dP}{d\zeta} \quad (35)$$

w = mass flux of water vapor per unit time and area

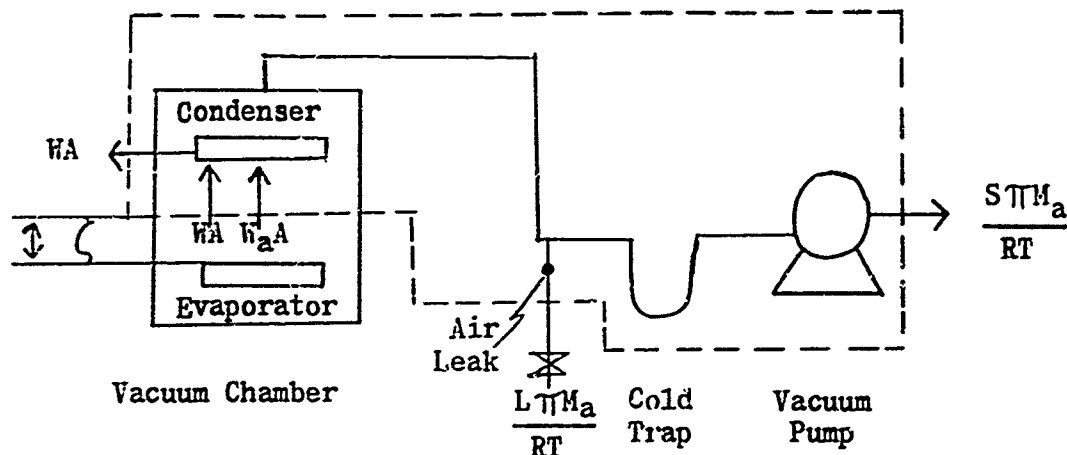
w_a = mass flux of air per unit time and area

P = partial pressure of water vapor at

ζ = distance from sample surface to condensing interface

M_a = molecular weight of air

An air material balance around the dryer shown below allows estimation of w_a in terms of speed of vacuum pump, S , and volume rate, L , at which air is leaked into the space bounded by dotted line:



Assumptions:

- (a) Steady state operation (w is equal to K).
- (b) Gas composition at any cross section is constant.
- (c) Air only is removed by vacuum pump (condenser and trap remove all water vapor).
- (d) Total pressure of system is insignificant.

Therefore, air flow at ζ is:

$$W_a = (S - L) \frac{\bar{\pi} M_a}{A R T} \quad \text{where } A = \text{area.} \quad (36)$$

Substituting this in Equation (35)

$$\frac{P}{\bar{\pi}} w = \frac{M \bar{W} (S - L) (\bar{\pi} - P)}{A R T} + \frac{D_v M \bar{W}}{R T} \frac{dp}{d\zeta} \quad (36a)$$

and

$$\frac{dp}{d\zeta} = \frac{dp}{d\zeta} \Big|_{\text{stag. air}} \left[1 - \frac{(\bar{\pi} - P) M \bar{W} (S - L)}{w A R T} \right] \quad (36b)$$

Equation (36b) may be integrated from p_i ($\zeta = 0$) to p_c ($\zeta = Z$) to obtain

$$w = \left(\frac{D_v M \bar{W} \bar{\pi}}{R T Z} \right) \left(\frac{P_i - P_c}{P'_{1m}} \right) - \frac{M \bar{W} (S - L) \bar{\pi}}{A R T} \quad (37)$$

In this equation

$$P'_{1m} = \ln \frac{P_c - P_i}{\left[\frac{R T A w p_c - M \bar{W} (S - L) \bar{\pi} P_c}{R T A w p_i - M \bar{W} (S - L) \bar{\pi} P_i} \right]} \quad \text{or a modified log mean air pressure.} \quad (38)$$

When $(S - L) = 0$, Equation (37) reduces to the equation for diffusion through stagnant air, Equation (34).

G. OVER-ALL MASS TRANSFER EQUATION

$$w = \frac{\frac{P_o^*}{\Gamma_s} - \frac{P_c^*}{\Gamma_c} - \frac{(S - L) Z P'_{1m}}{D_v A}}{\frac{1}{\Gamma_s \beta_s} + \frac{1}{\Gamma_c \beta_c} + \frac{L - x}{K_{fm} \beta_s d} + \frac{R T Z P'_{1m}}{M \bar{W} D_v \bar{\pi}}} \quad (39)$$

Assumptions:

- (a) Equations (21) and (21a) apply to mass transfer at subliming and condensing interface, respectively
- (b) Equation (28) applies to transport through dried cake
- (c) Equation (37) applies to vapor flow from dry cake to condenser

General Form: $w = \text{driving force} / \text{sum of resistance}$

Denominator includes resistances at two phase interfaces, through dried cake and between evaporator and condenser. Free molecular flow through cake layer and laminar flow to condenser have been assumed.

H. OVER-ALL MASS TRANSFER

Table III gives a summary of mass transfer resistances for all possible mechanisms of vapor movement. The relative importance of interphase mass transfer resistances in pure ice sublimation is shown for various total pressures and drying rates in Figure 5. This plot is based on theoretical calculations using Equations (21), (22), and (34) and assuming equilibrium water vapor pressure at condenser of 30 microns ($T_c^* = -50^\circ\text{C}$).

TABLE III. MASS TRANSFER RESISTANCES FOR POSSIBLE MECHANISMS OF VAPOR MOVEMENT

MASS TRANSFER STEP	RESISTANCES	
	FREE MOLECULAR OR TURBULENT	LAMINAR OR VISCOUS
Interphase I (ice to water vapor)	$\frac{1}{\Gamma_s \beta_s}$	$\frac{1}{\Gamma_s \beta_s}$
Dry cake (Subliming surface to cake surface)	$\frac{L-x}{K_{fm} \beta_{sd}}$	$\frac{L-x}{\frac{D_v MW \epsilon^2}{RT} + \frac{\psi P_o}{\mu}}$
Evaporator to condenser (cake surface to condenser)	$\frac{(a+b)Z}{8/3 K_{ab} \beta}$	$\frac{RTZ p'_{lm}}{MW D_v \pi}$
Interphase II (water vapor to ice)	$\frac{1}{\Gamma_c \beta_c}$	$\frac{1}{\Gamma_c \beta_c}$

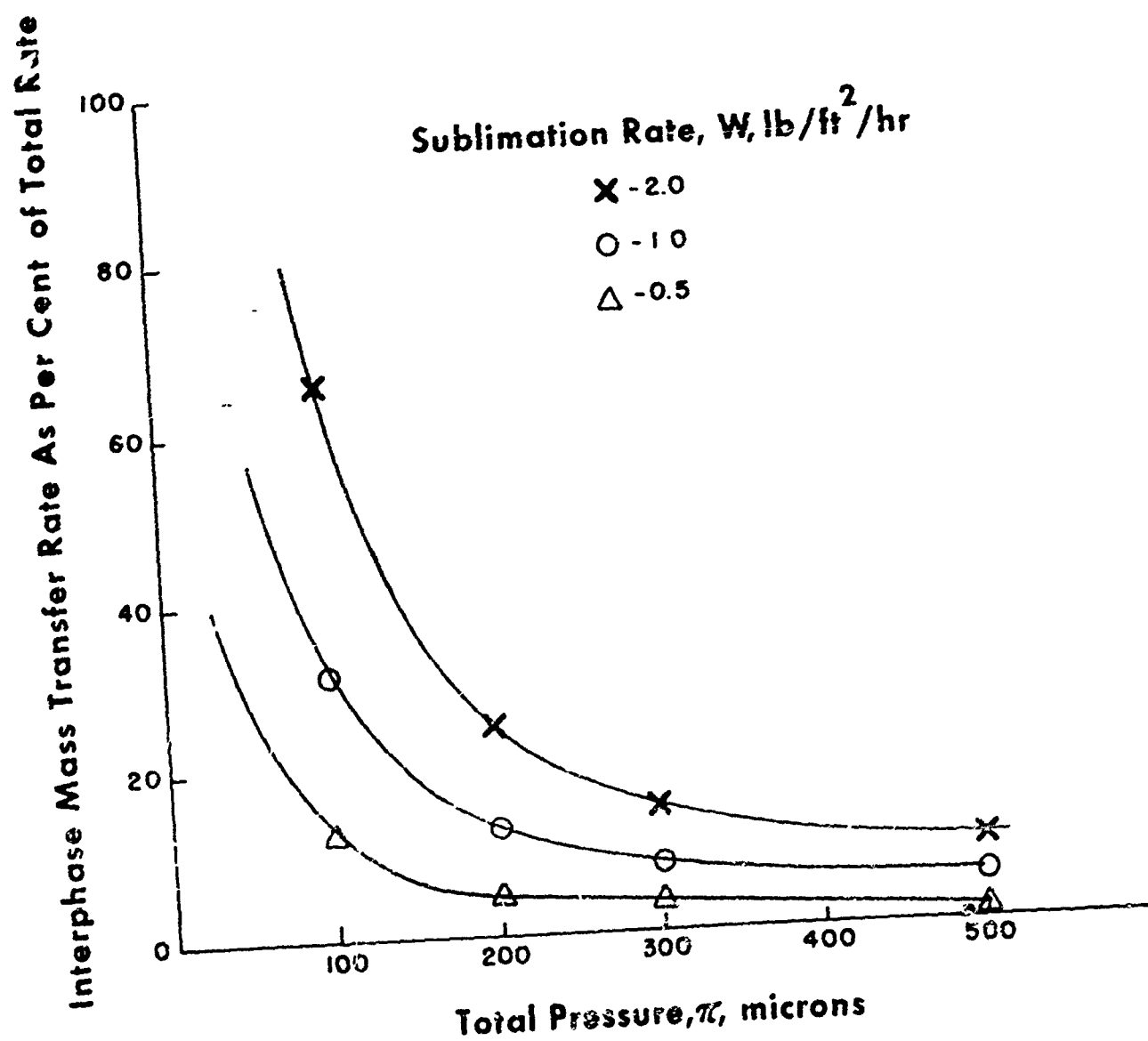


Figure 5. Interphase Mass Transfer Resistance in Sublimation of Ice as a Function of Total Pressure and Drying Rate.

Combination of Equation (19) for heat transfer and Equation (39) for mass transfer allows specification of temperatures in the ice cake during the constant-rate period, since t_s (or T_s^*) and P_s^* are an equilibrium pair. Effect of R_m , dry cake mass transfer resistance on subliming interface temperature, is shown in Figure 6.

The possible freeze-drying mass transfer driving forces and resistances are:

Driving Force	Resistance
Water vapor partial pressure difference between evaporator and condenser	Resistance to mass transport at the subliming interphase
Bulk flow of gas to pumping	Diffusional or flow resistance through dry cake above interface.
Transport by natural convection	Diffusional resistance to water vapor flow afforded by inert gas between evaporator and condenser
Entrainment of ice particles by moving gas stream	Resistance to interphase transfer at the condenser
Thermal diffusion	

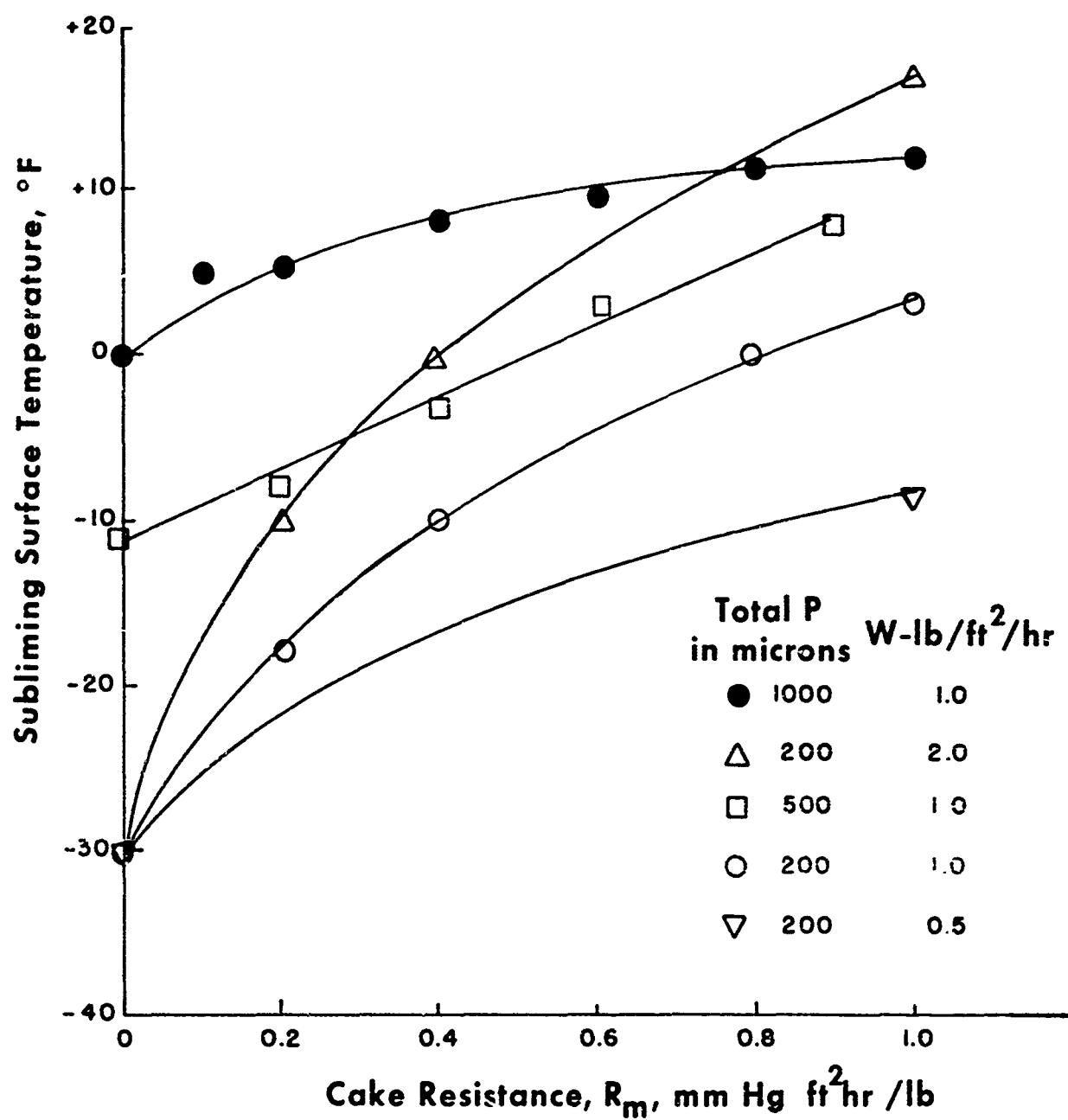


Figure 6. Dry-Cake Mass Transfer Resistance as a Function of Subliming Interface Temperature at Various Total Pressures and Mass Transfer Rates.

III. SAMPLE CALCULATION

Given a freeze dryer heated by conduction. The condenser-to-evaporator spacing is 0.5 foot. The evaporator area is 20 square feet. The refrigerated plate condensers which are maintained at -40°F are used to dry a one-inch-thick slab. The cake is in perfect thermal contact with the tray bottom. The material thaws at $+2^{\circ}\text{F}$. The maximum allowable temperature is 0°F .

Calculate:

1. Maximum possible drying rate and heat input if $P = 1000 \mu$
2. Calculate t_s or T_s^* and π if the total heat input is set at 16,300 Btu's per hour.

Neglect the dry cake resistance and assume that the thermal conductivity of the material is $1.5 \frac{\text{Btu}}{\text{hr ft } ^{\circ}\text{F}}$. The relationship between vapor pressure and temperature is the same as that for water, and the dryer efficiency is 75 per cent. Assume that the ice layer recedes uniformly into the cake.

Solution to 1.

Assume $\lambda_s W = 150 \text{ Btu/hr ft}^2$

$$t_i - t_s = \frac{\lambda_s W L}{K_i} = 150 \times \frac{1}{12} \times \frac{1}{1.5} = 8.3^{\circ}\text{F}$$

$$t_s = -8.3^{\circ}\text{F}$$

The corresponding drying rate is 0.130 lb/hr ft^2 or the calculated value of $\lambda_s W = (\lambda_s W)_{\text{calc}} = 0.130 \times 1220 = 156 \text{ Btu/hr ft}^2$. Since the first approximation was 150 Btu/hr ft^2 , we get

$$w = 0.130 \text{ lb/hr ft}^2 = 2.5 \text{ lb/hr.}$$

The true amount of heat added is

$$\frac{156 \text{ Btu/hr ft}^2}{0.75} = 208 \text{ Btu per hr ft}^2$$

or
$$208 \frac{\text{Btu}}{\text{hr ft}^2} \times 20 \text{ ft}^2 = 4160 \text{ Btu/hr.}$$

Solution to 2.

The value of the initial evaporating surface temperature, t_s , is again found using

$$t_s = \frac{-E_g L}{K_i} + t_i$$

$$t_s = \frac{0.75 \times 16,300}{1.5 \times 12 \times 20} + 0 = -34^\circ\text{F}$$

The vapor pressure of water at the subliming interface, P_o , is found

$$P_o = \frac{P_s^*}{\left(1 - \frac{W}{W_s^*}\right)} = \frac{P_s^*}{\left(1 - \frac{E_g}{P_s^*}\right)}$$

$$\beta_s = 118.5 \frac{\text{lb}}{\text{mm Hg ft}^2 \text{ hr}}$$

$$P_s^* = 0.140 \text{ mm Hg}$$

$$W_s^* = P_s^* \beta_s = 16.61 \text{ lb/ft}^2 \text{ hr}$$

$$W = \frac{E_g}{\lambda_s} = \frac{0.75 \times 16,300}{1220 \times 20} = 0.5 \text{ lb/ft}^2 \text{ hr}$$

$$\Gamma_s = 0.982$$

$$P_o = 0.140 \times (1 - 0.030) / 0.982 = 0.138 \text{ mm Hg.}$$

Similarly, the partial pressure of water at the condenser interface is computed to be 0.104 mm Hg. The total pressure is then calculated, neglecting bulk flow.

Rearranging,

$$P_{1m} = \frac{(0.8525) \left(\frac{426}{492}\right)^{2.334} (18)(0.138 - 0.104)}{(0.7302)(426)(0.5)(0.5)} = 0.005 \text{ mm Hg}$$

where the value of T has been taken as equal to the evaporator interface temperature. The total pressure is calculated from the log mean air pressure as follows:

$$P_{1m} = \frac{P_o - P_c}{\ln \frac{\pi - P_c}{\pi - P_o}} = \frac{0.138 - 0.104}{\ln \frac{\pi - 0.104}{\pi - 0.138}} = 0.005 \text{ mm Hg}$$

When this equation is solved for π , we get $\pi = -0.138 \text{ mm Hg}$.

Calculation of Drying Time

Calculate the drying time for the material in the above problem, if the critical moisture is known to be 1.5 lb H₂O/lb solid. Drying tests indicate that the rate is proportional to the moisture content for the entire falling-rate period. The initial solids content is 10 per cent; the bulk density of product is six lb/ft³, and the final moisture of the product is one per cent.

$$\theta = \theta_c + \theta_r = L \delta \left[\frac{\lambda_s (\bar{W}_o - \bar{W}_c)}{Eq} + \frac{1}{a} \ln \frac{\bar{W}_c}{\bar{W}_f} \right]$$

where "a" represents the slope of the rate vs moisture curve

Slope "a" is estimated by the relation

$$\bar{W}_c = \frac{Eq}{\lambda_s} = a \bar{W}_c$$

$$\text{or } a = \frac{Eq}{\lambda_s \bar{W}_c} = \frac{0.75 \times 16,300}{1220 \times 1.5 \times 2.0} = 0.33 \frac{\text{lb solid}}{\text{ft}^2 \text{ hr}}$$

After substitution into the equation for the total drying time, we obtain:

$$\theta = \frac{(1)(6)}{(12)} \left[\frac{(1220)(9.0-1.5)(20)}{(0.75)(16,300)} \right] + 3 \ln \frac{1.50}{0.01} = 15 \text{ hours.}$$

These sample calculations illustrate the use of the equations developed earlier, particularly Equations (19) and (39). The heat and mass transfer for conductive freeze-drying and the resistances connected with this type are considered in these relationships. The theoretical rates and times determined here were subsequently verified experimentally and good agreement was obtained.

IV. CONCLUSIONS

The principles of freeze drying are relatively recent in origin and are published in a variety of research areas. This report presents those principles governing freeze-drying rates of heat and mass transfer, and defines them in practical terms.

Some of these principles are:

(a) Primary factors which determine subliming interface temperatures are absolute total pressure and the mass transfer resistance of the dry solid above the evaporating surface. During the drying of substances containing solids the equilibrium vapor pressure is usually greater than the total pressure, but approaches it as the resistance to vapor flow through the residual solid above the interface decreases.

(b) The rate of drying is determined by the dryer efficiency and heat input to the evaporator. When drying slabs, the heat of sublimation must be conducted through the frozen layer. Therefore, heat input and slab thickness govern the rate of drying; the total pressure and conductivity determine the drying temperature level.

(c) The drying time per unit mass is greater for pellets than for slabs. The temperature control in a bed of pellets is more difficult. Even though drying occurs throughout the pellet bed, these two factors are related. Since the outer surface of any pellet dries first, the heat necessary for sublimation during later stages must pass through this dry exterior. Thermocouples tend to yield erroneous measurements unless properly located, and the heat input must be reduced earlier to avoid overdrying of the already dry exterior of any pellet, thus increasing the drying time. These same considerations preclude the use of radiant heat in a freeze-drying operation, because penetration of radiant energy into frozen samples is small and heat inputs must be kept low to prevent overdrying or uneven drying of the materials.

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GLOSSARY

All basic symbols, subscripts and superscripts used in this memorandum are defined here except for a few terms used only in limited contexts. The latter are defined in conjunction with their use. Where a symbol is shown as having more than one meaning, the context will indicate which definition is pertinent. The primary dimensions listed have the following meanings:

L = length
M = mass
 θ = time
T = temperature
F = force
Q = energy

<u>SYMBOL</u>	<u>MEANING</u>	<u>PRIMARY DIMENSIONS</u>
a	"slope" constant from rate <u>vs</u> moisture curve	$M/L^2\theta$
A	area	L^2
B	wall or film thickness	L
c	heat capacity	$Q/M(\Delta T)$
	concentration per unit mass	(unit)/M
C	cake conductivity	ML^2/θ
D	diffusion coefficient	L^2/θ
E	dryer efficiency	-
F	volume flow rate	L^3/θ
g	acceleration of gravity	L/θ^2
h	heat transfer coefficient	$Q/L^2\theta(\Delta T)$
J	"transitional" flow correction	-
k	thermal conductivity	$Q/L\theta(\Delta T)$
K	"geometric" factor for noncircular flow channel	

<u>SYMBOL</u>	<u>MEANING</u>	<u>PRIMARY DIMENSIONS</u>
l	molecular free path	L
L	cake thickness	L
	characteristic length	L
M or MW	molecular weight	M/mole
N	dimensionless number	
p	partial pressure of air	F/L^2
P	partial pressure of water	F/L^2
q	rate of energy transfer per unit area	Q/L^2
Q	heat content per unit volume	$Q/L^3\theta$
r	radius	L
R	outside radius	L
	gas constant	$FL/T(\text{mole})$
	heat or mass transfer resistances	$L^2(\Delta T)\theta/Q$ or $F\theta/M$
S	pumping speed	L^3/θ
t	condensed phase temperature	T
T	gas temperature (degrees absolute)	T
u	velocity	L/θ
V	volume	L^3
w	rate of mass transfer per unit area	$M/L^2\theta$
W	moisture content per mass of dry solid	M/M'
x	variable distance in ice or solid	L
Z	distance from cake surface to condenser	L

<u>SYMBOL</u>	<u>MEANING</u>	<u>PRIMARY DIMENSIONS</u>
β	function defined by Equation (21a)	θ/L
Δ	difference	
Γ	function defined by Equation (9)	
λ	latent heat per unit mass	Q/M
ν	kinematic viscosity	$M/L^4\theta$
μ	absolute viscosity	$M/L\theta$
Ω	ratio of liquid to ice density	
π	total pressure	F/L^2
ϕ	function defined by Equation (8)	
δ	bulk density of solid	M/L^3
ρ	true density	M/L^3
σ	accommodation coefficient	
θ	time	θ
ϵ	void fraction	L
ψ	cake permeability	L^2
ζ	variable surface from cake surface to gas phase	L

<u>SUBSCRIPT</u>	<u>MEANING</u>
a	air
b	container
c	condenser
	conduction
	constant rate
	critical
d	diffusion
e	effective
	evaporator
E	produced or generated
f	fusion
	falling rate
	final
h	heat transfer
i	ice
	ice-container interface gas at the cake surface
L	liquid or liquid phase
m	mass transfer
	maximum
	dry cake or material
O	initial or datum
	gas at the subliming interface orifice
R	refrigerant
s	solid
	sensible
	sublimation or subliming interface
v	vapor
x	value at position x
w	water
	latent heat
	heating medium
Gr	Grashof
Kn	Knudsen
Nu	Nusselt
Pr	Prandtl
Re	Reynolds
ln	log mean
bf	bulk flow
fm	free molecular

SUPERSCRIPTMEANING (\bar{x}) average or mean (x)

*

equilibrium value

FUNCTIONAL NOTATION

$$\operatorname{erfcy} = \frac{1}{2\pi^{1/2}} \int_0^x e^{-x^2} dx, \text{ the error integral}$$

$$-\operatorname{Ei}(-y) = \int_{-\infty}^{-y} \frac{e^y dy}{-y}, \text{ the exponential integral}$$